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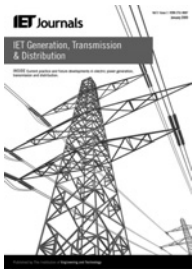

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Adaptive sequential importance sampling technique for short-term composite power system adequacy evaluation

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Abstract: The well-known sequential Monte Carlo simulation is a powerful tool to analyse short-term reliability of complicated composite power system. However, the corresponding computational burden is tremendous when applied to a highly reliable system. Aiming at improving the simulation efficiency, an adaptive importance sampling technology is proposed in this study. The proposed method, on the basis of component state duration sampling technique combined with cross-entropy method, is able to provide reliability evaluation of highly reliable non-homogeneous Markov system. The Weibull and lognormal distributions are considered to describe repair time of individual components comprising a system. Through the case studies conducted on a reinforced Roy Billinton reliability test system, it is validated that the proposed method is effective and of high efficiency and the efficiency gain against the crude sequential Monte Carlo simulation is robust to variation of load level and lead timescale. The proposed method is useful and efficient to timely monitor the system operating pressure under different lead timescales.

Nomenclature

Γ	system state transition path
T_L	lead time in hour
T_{Γ_i}	duration in hour of *i*th random realisation of Γ where all the states but the last one are normal system operating state
TTF	time to fail
TTR	time to repair
CSMC	crude sequential Monte Carlo
CSDS	component state duration sampling
SSTS	system state transition sampling
m_i	index number of state transition from working to failed state of component *i*
n_i	index number of state transition from failed to working state of component *i*
$M_{i,\Gamma}$	total number of state transitions of component *i* from working to failed state in path Γ within T_L.
$N_{i,\Gamma}$	Total number of state transitions of component *i* from failed to working state in path Γ within T_L
$\mathfrak{M}_{i,\Gamma}$	Total number of state transitions of component *i* from working to failed state in path Γ within T_Γ
$\mathfrak{N}_{i,\Gamma}$	Total number of state transitions of component *i* from failed to working state in path Γ within T_Γ
t_{f,m_i}	time to fail in hour associated with m_i^{th} transition
t_{r,n_i}	time to repair in hour associated with n_i^{th} transition
TC_i	accumulated time in hour of state transition simulation of component *i*, starting at $t=0$
τ	the shape parameter of Weibull distribution or the standard variance parameter of Gaussian distribution
ξ	the scale parameter of Weibull distribution or the mean parameter of Gaussian distribution
$S(\Gamma)$	real function describing the reliability metric of a realisation of path Γ, for example, loss of energy in MWh
co	coefficient of variance
$\mathbb{S}$	set of samples, each of which is simulated based on the CSDS method within T_Γ
$\mathbb{S}'$	set of samples, each of which is simulated based on the CSDS method within T_L
S_{IS}	set of samples, each of which is simulated based on the importance sampling technique within T_L
N	number of simulation runs of the CSMC method

1 Introduction

Power system short-term reliability evaluation is of utmost importance to power utilities confronted with power system operation variability and uncertainty arising from deregulation and renewable energy penetration [1]. Electric entities and independent system operators need to be carefully informed of system adequacy, so as to satisfy, with budget constraints, customer demand for higher service reliability and power quality. There exist many analysis tools for reliability evaluation, which generally fall into two categories, namely, analytical method [2, 3] and numerical simulation [4–6]. The Monte Carlo method is a well-known powerful tool to analyse the reliability of a complex system for its advantage of robustness to the dimension of the problem, capability to handle contingencies of all orders and simplicity in accommodating various power system models and operation modes. In terms of simulation mechanisms, there are two different sorts of Monte Carlo techniques: non-sequential Monte Carlo and sequential Monte Carlo. It is well recognised that, for complex system reliability evaluation, to calculate some reliability indices without bias, such as failure cost involving state duration, loss of load duration and loss of load expectation and so on entails sequential Monte Carlo method [7] in terms of its capability of simulating system stochastic responses chronologically. Nonetheless, both of these two sorts of techniques are confronted with a heavy simulation computational burden as a large number of system states must be sampled before the indices converge.

Tremendous work has been devoted to the improvement of computational efficiency of Monte Carlo methods, especially for the non-sequential Monte Carlo method, such as that reported in [4, 5, 8–11] and the references therein. Recently, an optimisation method proposed in [12] accelerates evaluation of the failure states of composite power system by eliminating redundant line flow constraints. This method can be combined with the existing state space sampling techniques to improve reliability evaluation efficiency. With respect to the crude sequential Monte Carlo (CSMC) method, a pseudochronological tool [13] and an algorithm for reliability simulation of equipment and systems by using a parallel computing environment [14] have been proposed. A sort of biasing technique is proposed in [15] for efficiently simulating system unavailability by uniformly distorting distribution of system state-transition time, along with discrete probability of selecting the component undergoing transition. However, the uniform distortion renders the occurrences of the state transition paths equally weighted, which should not be the case in terms of severity of the system failure state.

Another possible solution to overcome the inefficiency problem is to utilise the importance sampling technique. The crux of the matter when applying the importance sampling technique is to find the optimal change-of-measure. Recently, a technique combining the CSMC method and cross-entropy (CE) [16] method is proposed [17] to modify the chronological evolution of the system, as a result of which, the simulation efficiency and convergence properties are improved prominently for generating capacity reliability evaluation. With respect to the issue of sequential short-term reliability evaluation of composite power system, several troublesome difficulties need to be considered if the importance sampling technique is intended to be employed:

(1) Identification of a critical set consisting of system failure states, load-shedding for instance, involves complicated analysis. As a result, it is impossible to precisely recognise the critical set in advance unless a simulation study is conducted.
(2) High-dimensional parameter space may result in degeneration of the importance sampling method, viz. curse of dimensionality of likelihood ratios.
(3) Computation of a state-dependent cost usually involves remedial action analysis including power flow and/or optimal power flow analysis. Consequently, it should be avoided as much as possible to generate a huge pool of simulation samples.
(4) Stochastic operation characteristics of a realistic system would be described by the non-homogeneous Markov process where repair time of a comprising component is not exponentially distributed.

A critical issue for the importance sampling technique applied in reliability evaluation for a large composite power system is the curse of dimensionality which mainly results from a large number of parameters involved in the concerned problem. One possible approach as a compromise to handle the problem is to reduce the number of parameters by virtue of network reduction techniques [18–20]. In this paper, the curse of dimensionality will not be discussed. We focus on the other troublesome difficulties and propose a novel adaptive importance sampling technique for short-term sequential reliability evaluation of composite power system whose operational behaviour is represented by the non-homogeneous Markov process.

The remaining parts are organised as follows: issues associated with short-term reliability evaluation of composite power system are reviewed first in Section 2, followed by a discussion of the probabilistic models representing the component operation characteristics in short-term reliability analysis in Section 3, then a CSMC technique, namely component state duration sampling [21], adapted for system reliability evaluation within a fixed lead time is presented in Section 4. In the sequel section, theoretical foundation, basic idea and realisation steps of the proposed method are outlined, which is followed by a simple numerical example to illustrate necessity and unbiasedness of the proposed method applied on short-term reliability evaluation in Section 5.5. Finally, case studies are conducted on a reinforced Roy Billinton reliability test system (R-RBTS), through which it is shown that the proposed method is effective and of high efficiency.

2 Issues of short-term reliability evaluation of composite power systems

Short-term reliability evaluation can be used to measure the ability of a system withstanding unexpected system interruptions that will result from imminent probabilistic disturbances within a short time period in the near future. The considered short time period is the so-called lead time. The definition of the timescale of lead time varies from system to system because of different evaluation purposes. Generally used values of the lead time for power system short-time reliability evaluations range typically from 10 min to 10 h [22]. Empirical formulae defined in [23] to estimate the convergence time of loss of load probability can be used for lead timescale planning in practice according to specific system operating conditions. Reliability evaluation can be classified into adequacy analysis and security analysis, depending on the aspect of concern in case a disturbance occurs. In the short-term

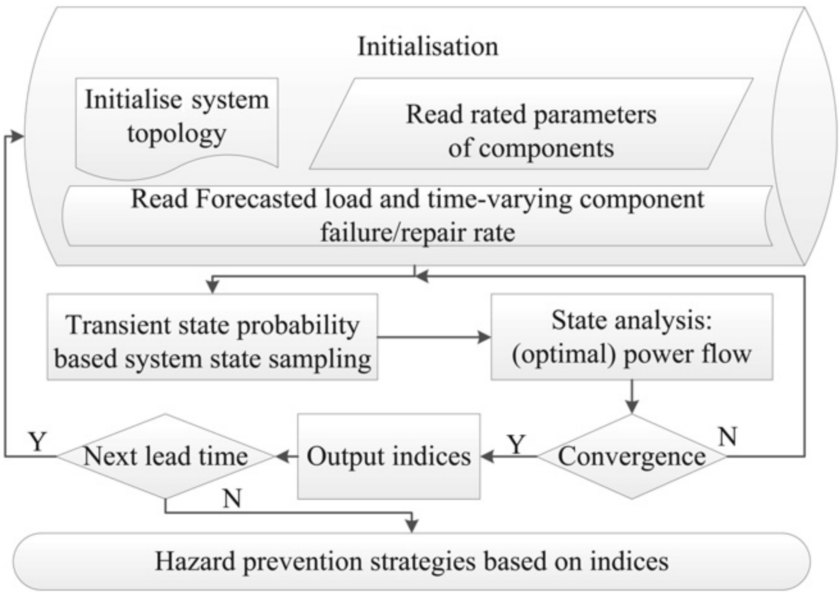

Fig. 1 *MCS-based analysis framework for short-term reliability evaluation*

adequacy evaluation, the sufficiency of facility online capacity to meet the system load demand in static system conditions is the essential concern. It is assumed that the system subjected to a disturbance can automatically restore or be manually controlled to be stable, that is to say, such events as time-dependent bus voltage collapse and generator rotor angle instability would not be triggered by the perturbation, and remedial actions such as load shedding, transformer tap changing and/or re-dispatching generator outputs can be thereof conducted anyway in time to bring an occurred abnormal state back into a stable equilibrium point. Owing to the short time-scale considered in the short-term reliability evaluation, the failure rate of a transmission or distribution line is not a steady-state value, but a function of the environment it is exposed to, for instance, it can be much higher in adverse weather than in normal weather. In addition, such events as scheduled outages, planned maintenance and postponed failures [24] should not be considered and the forecasted load level could be regarded as constant, or random but complying with a certain distribution, for example, Gaussian distribution.

In terms of the complexity of composite power systems, the sequential Monte Carlo method is popularly resorted to for the short-term reliability evaluation of the features of easy implementation, ergodicity and robustness to system dimension. The necessity of the sequential Monte Carlo method applied to short-term reliability studies is illustrated in Section 5.5. A flowchart of the analysis based on the Monte Carlo method for the short-term composite power system reliability evaluation is schematically shown in Fig. 1. Once the data of the system configuration and parameters of the components as well as forecasted load curve within each lead time are ready, Monte Carlo Simulation is conducted to simulate the system states in a non-sequential or sequential manner according to transient probabilities or transient state transition rates of the components, and each drawn state is identified as either a successful state or a failed state by virtue of the power flow and/or the optimal power flow analysis. The stop criteria can be defined on a limitation of the number of simulation runs or the co [25] of a certain index. The evaluation task is repeated for each lead time one by one, where the data relevant to system operational conditions will be necessarily reloaded before a new evaluation task begins. The evaluation process ceases until hazard prevention strategies based on the released indices become necessary.

3 Component probabilistic function models

It is a common practice to assume that both time-to-fail's (TTFs) and time-to-repair's (TTRs) of components comprising a system are exponentially distributed for mathematical convenience in reliability evaluation. As a result, the system is a homogeneous Markov system. However, because of a time-varying operation condition, the failure rate of a component could not be represented by its average value as used in the long-term evaluation; moreover, component repair time is validated by many researches to be occasionally illustrated as some other distributions [26–28], such as lognormal or Weibull. Consequently, the sequential system state-transition can be considered as the non-homogeneous Markov process. In this paper, within the interval of each lead time, the failure rate of a component is assumed to be a constant, and additionally, each component is assumed to operate independently in its useful life period and can be restored to an 'as bad as old state' by assumption of minimal corrective maintenance upon a failure. [In preventive maintenance policy studies, it is commonly assumed for minimal repair models that the system failure rate function is not disturbed by any minimal corrective maintenance, that is, the system could be regarded 'as bad as old' after minimal corrective maintenance [29].] That is, the individual component operational process is regenerative, hence the semi-Markov process can be used to model the probabilistic characteristics of the whole system operation [30]. The semi-Markov process is an extension of the homogeneous Markov process. For a Markov process, if the transition time of at least one of its states does not follow an exponential distribution, it is called the semi-Markov process. One general method to simulate a system state transition path, as we shall discuss in Section 4, is to independently draw state durations of each comprising

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component by using the CSMC method with TTF and/or TTR probability distributions of those components.

In this paper, both Weibull and lognormal distributions are considered for component TTRs without loss of generality. The probability density functions (PDFs) of the two distributions are uniformly denoted by $f_r(t, \xi, \tau)$ for brevity and expressed as follows

$$f_r(t, \xi, \tau) = \begin{cases} \frac{\tau}{\xi}(t/\xi)^{\tau-1}\,\mathrm{e}^{-(t/\xi)^{\tau}} & \text{for Weibull} \\ \frac{1}{\sqrt{2\pi}\tau t}\mathrm{e}^{-(1/2)\left((\ln t-\xi)/\tau\right)^2} & \text{for log normal} \end{cases} \tag{1}$$

where ξ and τ represent either the scale and shape parameters in the case of Weibull distribution, or in the case of lognormal distribution, the mean and standard deviation parameters of associated normal distribution. In addition, component TTF is assumed to follow the exponential distribution with λ representing the corresponding failure rate.

4 Component state duration sampling technique

There exist two sorts of well-developed CSMC methods for reliability evaluation: the system state-transition sampling (SSTS) [31] method and component state duration sampling (CSDS) method [21]. The SSTS method works on conditions where the system states are Markov states and the duration distributions of all possible states are known in advance. It is easy to satisfy the condition by assuming the state duration of each comprising component to be exponentially distributed. This results in that each possible system state duration also follows exponential distribution, and the system state transition rate is thereof the sum of all the component state transition rates relevant to the current states of the components residing in this system state. In the situation that the component TTR follows the Weibull distribution, an aggregate Weibull [30] approach based on the SSTS method could be utilised to deal with the sequential simulation for the non-homogeneous system if the concerned system failure events are not rare; nevertheless, the resulting duration distribution of a system state is too complicated for the importance sampling technique to tackle in the case of rare system failure events. By contrast, the CSDS method is more competent to accomplish the simulation task, with no need to deduce the probability distributions of system state transition and transition time but the state and duration distributions of individual comprising component, which can be acquired more straightforwardly.

Supposing a system composed of binary Markovian state components with working (W) and failed (F) states, respectively, (initially in W state), all the components operate independently, the CSDS procedure for the system reliability evaluation within a fixed lead time T_L is schematically shown in Fig. 2 and the corresponding steps of the procedure read as follows:

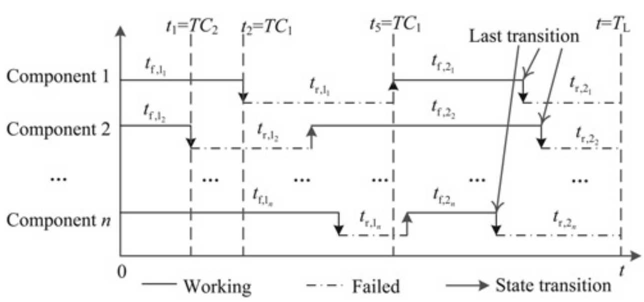

Fig. 2 *Schematic of the CSDS method*

Step 1: Initialise the count of the number of simulation, $j = 1$.
Step 2: Initialise the simulation starting time, $t = 0$, state transition index number of W state to F state, $m_i = 0$, index number of F state to W state, $n_i = 0$ with $i \in 1, \ldots, \Omega$ where Ω represents the number of components comprising the system.
Step 3: Update $m_i := m_i + 1$ and randomly produce a TTF for component i and record it as t_{f,m_i}, $t_{\mathrm{f},m_i} = \min\left(T_L, -\ln\left(u_{m_i}\right)/\lambda_i\right)$, $i \in 1, \ldots, \Omega$, where u_{m_i} is a randomly sampled variable distributed uniformly on (0, 1), and let TC_i represent the accumulated time of state transition simulation of component i and $\mathrm{TC}_i := t_{\mathrm{f},m_i}$, $i \in 1, \ldots, \Omega$.
Step 4: Let the duration of the current system state at t be equal to $\mathrm{TC}_k - t$ where $k = \arg\min i \leq \Omega\{\mathrm{TC}_i\}$. The system state at t and the resulting state duration $\mathrm{TC}_k - t$ combined with a forecasted load level at the current time point t are used for the state reliability metric calculation, for example, energy unserved in megawatt per hour over $\mathrm{TC}_k - t$. Renew $t := \mathrm{TC}_k$. If $t = T_L$, sum all the calculated quantities over T_L to obtain a reliability metric realisation of random system state transition path, denoted by $S(\Gamma_j)$ with $\Gamma_j \in \Gamma_1, \Gamma_2, \ldots, \Gamma_N$ defined as the random system state transition path within T_L where N is the cardinality of the pool of simulated realisations of path Γ, and then go to Step 5; otherwise, go to Step 6.
Step 5: If a pre-defined convergence condition, such as co for the support of $S(\Gamma)$ is satisfied, stop the simulation; otherwise, $j := j + 1$, go to Step 2. The co related with the support of $S(\Gamma)$ can be calculated by

$$\mathrm{co} = \frac{\sqrt{\sum_{i=1}^{N}\left(E[S(\Gamma)] - S\left(\Gamma_i\right)\right)^2}}{\sqrt{N(N-1)}E[S(\Gamma)]} \tag{2}$$

where $E(\cdot)$ denotes the expectation operator, and according to the central limit theorem, $E[S(\Gamma)] \simeq \frac{\sum_{i=1}^{N} S(\Gamma_i)}{N}$ when N is large enough.
Step 6: If the state of component k is W just before t, change the state to F, and go to Step 7; otherwise, change the state to W, and go to Step 8.
Step 7: Update $n_k := n_k + 1$ and produce a TTR denoted by t'_{r,n_k} according to the TTR distribution of component k by means of the inverse-transform technique. Let $t_{r,n_k} = \min\left(t'_{r,n_k}, T_L - \mathrm{TC}_k\right)$, and renew $\mathrm{TC}_k = \mathrm{TC}_k + t_{\mathrm{r},n_k}$, then return to step 3.
Step 8: Update $m_k := m_k + 1$ and produce a TTF denoted by t'_{f,m_k} according to the TTF distribution of component k by means of the inverse-transform technique. Let $t_{\mathrm{f},m_k} = \min\left(t'_{\mathrm{f},m_k}, T_L - \mathrm{TC}_k\right)$, and renew $\mathrm{TC}_k = \mathrm{TC}_k + t_{\mathrm{f},m_k}$, then return to step 3.

It can be observed from Fig. 2 that the CSDS procedure finally splits a lead time into smaller time intervals with variant lengths, for each small time interval the begin or end point is the time instant just one component of the whole system changes its state, whereas within each time interval, the system resides in the state coming up at the begin time point without variation. Moreover, it is worth underlining that if a component initially failed at the beginning of the simulation, historical duration of the failed state of the component is needed to be subtracted from its firstly drawn TTR [6].

5 CE-based adaptive sequential importance sampling

5.1 CE method

Importance sampling has emerged in the literature as a powerful tool to reduce the variance of an estimator, which in the case of rare event estimation also means increasing the occurrence rate of the rare event. If the expected value $E[S(x)]$ of a random variable x with density g is to be computed then, the importance sampling method estimates

$$E\big[S(x)\big] = S(x)\frac{g(x)}{h(x)}h(x)\mathrm{d}x$$

by

$$E\big[S(x)\big] = \frac{1}{n}\sum_{i=1}^{n} S(x_i)\frac{g(x_i)}{h(x_i)} \tag{3}$$

where $x_1, \ldots, x_n$ are independently and identically distributed copies of x, which are drawn from $h(x)$. The aim is to find a proper $h(x)$ with which the importance sampling estimator has small variance, specifically, if $h(x)$ is taken as (4), then the importance sampling estimator (3) has zeros variance. The optimal change of measure, namely zero-variance PDF, for general importance sampling problems can be given as

$$h(x_i) = g(x_i)\frac{S(x_i)}{E\big[S(x)\big]} \quad i \in \{1, 2, \ldots, n\} \tag{4}$$

By substituting (4) into (3), it can be noted that we need nothing but only one simulation to obtain $E[S(x)]$ (which means the sample has zero variance); however, the exact $h(x)$ cannot be known before simulation, or even does not exist for complex problems. Thus, $h(x)$ needs to be estimated beforehand. The existing methods to estimate $h(x)$ can be classified into parametric and non-parametric methods [32]. The CE method falls into parametric method combined with iterative sample learning mechanism. The basic idea of CE is to find a surrogate PDF for $h(x)$ which is optimised to minimise the Kullback–Leibler distance from $h(x)$ [16]. Compared with other metrics quantifying the distance between the two PDFs, CE is more applicable to process the exponential distribution family [33].

Provided that β represents a vector of the original parameters of a density function $g(x;\beta)$ which is related with a problem concerned, let $g(x;\alpha')$ represent a PDF stemming from the same distribution family from which $g(x;\beta)$ comes. Then, the parameter vector α' can be iteratively obtained through (5) by minimising the Kullback–Leibler distance between $g(x;\alpha')$ and the problem related zero-variance PDF

$$\alpha'^{(n+1)} = \arg\max_{\alpha'} \times \left[\frac{1}{N}\sum_{i=1}^{N} S\Big(x_i^{(n)}\Big)\frac{g\Big(x_i^{(n)};\beta\Big)}{g\Big(x_i^{(n)};\ \alpha'^{(n)}\Big)}\ln g\Big(x_i^{(n)};\ \alpha'\Big)\right] \tag{5}$$

where the superscript (n) denotes the number of iterations, $\boldsymbol{\alpha}'^{(n)}$ is a vector of nth distorted version of β and generally $\alpha'^{(1)} \equiv \beta$, N is a pre-defined total number of simulation runs, $x_i^{(n)}$ is a sample from $g(x;\alpha'^{(n)})$. In most practical problems $S(\cdot) \geq 0$, thus as long as $\ln g(x;\alpha')$ is convex and differentiable with respect to α', the problem of solving (5) will be equivalently reduced to solve the following equation

$$\sum_{i=1}^{N} S\Big(x_i^{(n)}\Big)\frac{g\Big(x_i^{(n)};\beta\Big)}{g\Big(x_i^{(n)};\ \alpha'^{(n)}\Big)}\frac{\partial \ln g\Big(x_i^{(n)};\ \alpha'\Big)}{\partial \alpha'} = 0 \tag{6}$$

The nth iterative solution of (6) is just the general estimator for optimal distorted parameters of the PDF from the viewpoint of CE.

5.2 Ideas underlining the proposed method

According to (6), $g(\cdot)$ is a problem-specific function which is critical for applying the cross-entropy technique. In terms of the short-term reliability evaluation problem for a composite power system within a fixed lead time, $g(\cdot)$ represents the likelihood of random system transition path within a fixed lead time T_{L}. Provided that the components comprising the system operate independently and each with binary Markovian states, TTFs and TTRs of any individual component within T_{L} are also independent. Let $P_{\mathrm{W}i}(\cdot)$ represent the probability mass function of state transition number of component i from W to F and $P_{\mathrm{F}i}(\cdot)$ represent that of component i from F to W within T_{L}. Given each component initially operates in W state, then, the likelihood $g(\Gamma;\beta)$ can be constructed as the following equation

$$g(\Gamma;\beta) = \prod_{i=1}^{\Omega}\sum_{m_i=0}^{\infty}\sum_{n_i=0}^{\infty} J_i \times \big(\Gamma;\beta|m_i, n_i\big)P_{\mathrm{W}i}\big(m_i\big)P_{\mathrm{F}i}\big(n_i\big) \tag{7}$$

where $J_i(\Gamma;\beta/m_i, n_i)$ is a conditional PDF of the ith component state transition path conditioned on transition number m_i and n_i within T_{L} and can be expressed as the following equation

$$J_i\big(\Gamma;\beta|M_{i,\Gamma}, N_{i,\Gamma}\big) = \prod_{m_i=1}^{M_{i,\Gamma}}\lambda_i \mathrm{e}^{-\lambda_i t_{f,m_i}}\prod_{n_i=1}^{N_{i,\Gamma}} f_r\Big(t_{r,n_i}, \xi_i, \tau_i\Big)\kappa_i \times \left(T_{\mathrm{L}} - \sum_{m_i=1}^{M_{i,\Gamma}} t_{\mathrm{f},m_i} - \sum_{n_i=1}^{N_{i,\Gamma}} t_{r,n_i}\right) \tag{8}$$

where β represents the set of original λ, ξ and τ associated with all the components, $M_{i,\Gamma}$ is the number of state transitions of component i from W to F related with Γ within T_{L} and $N_{i,\Gamma}$ is that of component i from F to W. It is noteworthy that in order to reduce variance, the

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complement cumulative probability rather than PDF of the last state duration beyond T_L for component i [34] needs to be taken into account, which is reflected as $\kappa_i(t)$

$$\kappa_i(t) = \begin{cases} e^{-\lambda_i t} I_{W,i}(T_L) = 1 \\ \bar{F}_r(t, \xi_i, \tau_i) I_{W,i}(T_L) = 0 \end{cases} \tag{9}$$

where $\bar{F}_r(t, \xi_i, \tau_i) = \int_t^{-\infty} f_r(\delta, \xi_i, \tau_i) \mathrm{d}\delta$ and $I_{W,i}(T_L)$ is an indicator function which equals to 1 if component i is in W state at T_L, or otherwise 0.

As each Γ_k, $k \in 1, 2, \ldots, N$ drawn by employing the CSDS technique definitely yields M_{i,Γ_k} and N_{i,Γ_k}, for all $i \in 1, \ldots, \Omega$. Then

$$g(\Gamma_k; \beta) = \prod_{i=1}^{\Omega} J_i\left(\Gamma_k; \beta | M_{i,\Gamma_k}, N_{i,\Gamma_k}\right) P_{Wi}\left(M_{i,\Gamma_k}\right) P_{Fi}\left(N_{i,\Gamma_k}\right) \tag{10}$$

If (10) was crudely substituted into (6), the resulting equation is too complicated to be solved. However, if M_{i,Γ_k} and N_{i,Γ_k} ($i \in 1, \ldots, \Omega$, $k \in 1, \ldots, N$) are not intended to be optimised, that is, regarded as constant, then when substituting (10) into (6) we can obtain a simplified version (11) with M_{i,Γ_k} and N_{i,Γ_k} eliminated.

$$\sum_{k=1}^{N} S'\left(\Gamma_k^{(n)}\right) \frac{\partial \ln J\left(\Gamma_k^{(n)}; \alpha'\right)}{\partial \alpha'} = 0 \tag{11}$$

where

$$S'\left(\Gamma_k^{(n)}\right) = S\left(\Gamma_k^{(n)}\right) \frac{J\left(\Gamma_k^{(n)}; \beta\right)}{J\left(\Gamma_k^{(n)}; \alpha'^{(n)}\right)}$$

Now, the paradigm to solve the optimal distorted parameters used for CE-based importance sampling method has been constructed. However, there is still another critical problem about how to efficiently obtain $\Gamma_k^{(n)}$ satisfying $S'\left(a_k^{(n)}\right) > 0$ so as to make it feasible to update α' according to (11). Our idea is as follows: in the first iteration, $n = 1$, we draw a moderate number of system state transition paths according to the original parameters of the component reliability models, where the predefined stop criterion to form each drawn path is that any system failure state is hit in the sampling process. Cost for each path is uniformly defined as the unit value, that is, $S\left(\Gamma_k^{(1)}\right) \equiv 1$, $k = 1, \ldots, N$. Hence, the duration of each drawn path, denoted byT_{Γ_k}, is random. We select drawn paths with relatively short duration and substitute them together with the corresponding $S\left(\Gamma_k^{(n)}\right)$ into (11), then the resulting solved parameters can be used to simulate $\Gamma_k^{(2)}$. Repeating the procedure $n - 1$ times leads to the observation that the system failure event is not rare any more within the concerned lead time. Consequently, in the next stage, a sufficiently large pool of Γ leading to $S'\left(\Gamma_k^{(n)}\right) > 0$ can be sampled efficiently, and the obtained costs compensated by a proper likelihood ratio are finally synthesised to yield the expectation quantity of interest according to the philosophy of the importance sampling technique. It can be noted that, in order to solve (11) in the two stages, two stop criteria are defined to assist in simulating the needed system state transition paths, respectively: Criterion (1) hitting any system failure state, and Criterion (2) reaching the end of a specific lead time. For the sake of clarity, in the first simulation stage under Criterion (1), the duration of Γ_k, T_{Γ_k}, should replace T_L in (8) to update the relevant parameters. Thus, the resulting likelihood can be expressed as follows

$$\hat{J}_i(\Gamma; \beta) = \prod_{i=1}^{\Omega} \prod_{m_i=1}^{\mathfrak{M}_{i,\Gamma}} \lambda_i e^{-\lambda_i t_{f,m_i}} \prod_{n_i=1}^{\mathfrak{N}_{i,\Gamma}} f_r\left(t_{r,n_i}, \xi_i, \tau_i\right) \times \kappa_i\left(T_{\Gamma_i} - \sum_{m_i=1}^{\mathfrak{M}_{i,\Gamma}} t_{f,m_i} - \sum_{n_i=1}^{\mathfrak{N}_{i,\Gamma}} t_{r,n_i}\right) \tag{12}$$

where $\mathfrak{M}_{i,\Gamma}$ and $\mathfrak{N}_{i,\Gamma}$ are the quantity of the component i state transitions from W to F and F to W related to Γ within T_Γ, respectively.

5.3 Optimal change of measure from the viewpoint of CE

Substitute (1), (9) and (12) into (11), the final nth iteration yields $\lambda_{i_{\text{opt}}}$, $\tau_{i_{\text{opt}}}$, $\xi_{i_{\text{opt}}}$ as follows under Criterion (1)

$$\lambda_{i_{\text{opt}}} = \frac{\sum_{k=1}^{ST^{(n)}} \mathfrak{M}_{i,k} S'\left(\Gamma_k^{(n)}; \beta, \alpha_{CE}^n\right)}{\sum_{k=1}^{ST^{(n)}} S'\left(\Gamma_k^{(n)}; \beta, \alpha_{CE}^n\right) \sum_{m_i=1}^{\mathfrak{M}_{i,k} + I_{W,i}\left(T_{\Gamma_k}\right)} t_{f,m_i}} \tag{13}$$

where $ST^{(n)}$ stands for the quantity of simulated paths in nth iteration, and $\mathfrak{M}_{i,k} = \mathfrak{M}_{i,\Gamma_k}$.

In the case of Weibull distribution, $\mu_{i_{\text{opt}}}$ and $\sigma_{i_{\text{opt}}}$ can be obtained by solving the non-linear-equation set as (14) with Newton–Raphson method (see (14))
where $I_{F,i}(\cdot) = 1 - I_{W,i}(\cdot)$and $\mathfrak{N}_{i,k} = \mathfrak{N}_{i,\Gamma_k}$.

$$1\begin{cases} \sum_{k=1}^{ST^{(n)}} S'\left(\Gamma_k^{(n)}; \beta, \alpha_{CE}^{(n)}\right) \\ \times \left\{ \sum_{n_i=1}^{\mathfrak{N}_{i,k}} \frac{1}{\tau_{i_{\text{opt}}}} + \ln \frac{t_{r,n_i}}{\xi_{i_{\text{opt}}}} \left[1 - \left(\frac{t_{r,n_i}}{\xi_{i_{\text{opt}}}}\right)^{\tau_{i_{\text{opt}}}}\right] - I_{F,i}(T_{\Gamma_k}) \left(\frac{t_{r,\mathfrak{N}_{i,k}+1}}{\xi_{i_{\text{opt}}}}\right)^{\tau_{i_{\text{opt}}}} \ln\left(\frac{t_{r,\mathfrak{N}_{i,k}+1}}{\xi_{i_{\text{opt}}}}\right) \right\} = 0 \\ \sum_{k=1}^{ST^{(n)}} S'\left(\Gamma_k^{(n)}; \beta, \alpha_{CE}^{(n)}\right) \\ \times \left[\sum_{n_i=1}^{\mathfrak{N}_{i,k}} -\frac{1}{\xi_{i_{\text{opt}}}} + t_{r,n_i}^{\tau_{i_{\text{opt}}}} \xi_{i_{\text{opt}}}^{\left(-\tau_{i_{\text{opt}}}-1\right)} + I_{F,i}\left(T_{\Gamma_k}\right) t_{r,\mathfrak{N}_{i,k}+1}^{\tau_{i_{\text{opt}}}} \xi_{i_{\text{opt}}}^{\left(-\tau_{i_{\text{opt}}}-1\right)} \right] = 0 \end{cases} \tag{14}$$

In the case of lognormal distribution, as the close-form cumulative distribution function of normal distribution does not exist, a polynomial approximation [35] is used instead

$$\bar{F}(z) = f(z)\sum_{i=1}^{5}\frac{b_i}{(1+rz)^i} \tag{15}$$

where $\bar{F}(z) = \int_z^{\infty} f(\pi)d(\pi), f(z) = 1/\sqrt{2\pi}\exp(-z^2/2)$, $r = 0.2316419$, $b_1 = 0.31938153$, $b_2 = -0.356563782$, $b_3 = 1.781477937$, $b_4 = -1.821255978$ and $b_5 = 1.330274429$. Then, substituting (1) and (8) along with (15) into (6), $\tau_{i_{\rm opt}}$ and $\xi_{i_{\rm opt}}$ can be obtained by solving the non-linear-equation set as (16) with the Newton–Raphson method (see (16))

5.4 Procedure of adaptive sequential importance sampling

As mentioned above, the failure of a highly reliably system is a rare event with regard to small component failure rate, consequently, the concerned sample of the system state transition path with $S(\Gamma_i) > 0$ is scarcely captured within a short lead time. To address this issue, adaptive sequential importance sampling proposed in this paper is introduced in as follows:

Step i: Initialise the parameters including: $n = 1$, the lead time $T_{\rm L}$, a percentage variable $\rho \in [0.01, 0.1]$ and a smoothing parameter $\omega = 0.1$. $\alpha_{\rm CE}^{(n)} = K(\beta)$ where $K(\cdot)$ is a function defined for initialisation of the parameters used for sampling, which will be explained later.
Step ii: Independently simulate a set ($\mathbb{S}$) of system state transition paths via a similar procedure as the CSDS with $\alpha_{\rm CE}^{(n)}$, except that

The stop criterion for each drawn path is that a system failure state is firstly triggered, with the epoch $\forall\Gamma_i \in \mathbb{S}$ denoted by T_{Γ_i}.
In Step 4, $S(\Gamma_i) \triangleq 1$, $\forall\Gamma_i \in \mathbb{S}$, is further multiplied by $\hat{J}(\Gamma_i;\beta)/\hat{J}\left(\Gamma_i;\alpha_{\rm CE}^{(n)}\right)$ to yield $S_t'(\Gamma_i)$.

Step iii: Arrange the set of T_{Γ_i} in ascending order to constitute a new epoch set denoted by $\Upsilon = \{t_1', t_2' \ldots\}$. To compute $\alpha_{\rm CE}^{(n+1)}$ according to (13), (14) and (16), gather $S_t'(\Gamma_i)$ with corresponding Γ_i s satisfying $t_{\Gamma_i} \le t_n'$ where $n = \sup\{i, i \le \rho|\Upsilon|\}$ with $|\cdot|$ representing the cardinality operator. If the parameters $\lambda_b^{(n+1)}$, $\xi_b^{(n+1)}$ and $\tau_b^{(n+1)}$ are found to be zero or infinite, then they will be left unchanged as their last iteration solution in $\alpha_{\rm CE}^{(n)}$. Sequentially, $\alpha_{\rm CE}^{(n)}$ is further smoothly updated by (17). If $t_n' > T$, $n =: n+1$ and return to Step ii; otherwise, continue

$$\alpha_{\rm CE}^{(n+1)} := (1-\omega)\alpha_{\rm CE}^{(n+1)} + \omega\alpha_{\rm CE}^{(n)} \tag{17}$$

Step iv: Independently simulate a set ($\mathbb{S}'$) of system state transition paths via a similar procedure as the CSDS with $\alpha_{\rm CE}^{(n)}$, except that in Step 4, $S(\Gamma_i)$ is further multiplied by $J(\Gamma_i;\beta)/J\left(\Gamma_i;\alpha_{\rm CE}^{(n)}\right)$ to yield $S_{\rm IS}'(\Gamma_i)$. Compute $\alpha_{\rm CE}^{(n+1)}$ according to (13), (14), (16) and (17).
Step v: Independently simulate a set ($\mathbb{S}_{\rm IS}$) of system state transition paths via a similar procedure as the CSDS with $\alpha_{\rm CE}^{(n+1)}$ except that in Step 4, $S(\Gamma_i)$ is further multiplied by $J(\Gamma_i;\beta)/J\left(\Gamma_i;\alpha_{\rm CE}^{(n+1)}\right)$ to yield $S_{\rm IS}'(\Gamma_i)$. Finally, the desired expected value is obtained through $E[S(\Gamma)] = \sum_{i=1}^{|\mathbb{S}_{IS}|} S_{\rm IS}'(\Gamma_i)/|\mathbb{S}_{\rm IS}|$, in the meanwhile, a predefined convergency condition, such as co [see (2)] or a limited number of simulation runs, is respected.

It is worth underlining that in principle, $K(\cdot)$ can be arbitrarily defined as long as $J\left(\Gamma;\alpha_{\rm CE}^{(1)}\right) \ne 0$ can hold $\forall\Gamma$, $S(\Gamma)J(\Gamma;\beta) \ne 0$. With regard to the composite power system short-term reliability evaluation, it is intuitive that a high component failure rate results in high possibility of system failure event, thus, Step i can be expedited by simply defining $K(\cdot)$ as an operator to magnify the original component failure rates by an equal multiple.

5.5 Simple numerical example

5.5.1 Discussions of sequential Monte Carlo applied for short-term reliability evaluations:

To appreciate the differences between analytical, non-sequential

$$\begin{cases} \displaystyle\sum_{k=1}^{{\rm ST}^{(n)}} S'\left(\Gamma_k^{(n)};\beta,\alpha_{\rm CE}^{(n)}\right)\left[\sum_{n_i=1}^{\mathfrak{N}_{i,k}}\frac{lnt_{{\rm r},n_i}-\xi_{i_{\rm opt}}}{\tau_{i_{\rm opt}}^2} + I_{{\rm F},i}\left(T_{\Gamma_k}\right)\cdot\right. \\ \left.\times\left(\frac{lnt_{{\rm r},\mathfrak{N}_{i,k}+1}-\xi_{i_{\rm opt}}}{\tau_{i_{\rm opt}}^2} + \frac{\sum_{j=1}^{5}\dfrac{jb_jr/\tau_{i_{\rm opt}}}{\left[1+r\left(lnt_{{\rm r},\mathfrak{N}_{i,k}+1}-\xi_{i_{\rm opt}}\right)/\tau_{i_{\rm opt}}\right]^{j+1}}}{\sum_{j=1}^{5}\dfrac{b_j}{\left[1+r\left(lnt_{{\rm r},\mathfrak{N}_{i,k}+1}-\xi_{i_{\rm opt}}\right)/\tau_{i_{\rm opt}}\right]^{j}}}\right)\right] = 0 \\ \displaystyle\sum_{k=1}^{{\rm ST}^{(n)}} S'\left(\Gamma_k^{(n)};\beta,\alpha_{\rm CE}^{(n)}\right)\left[\sum_{n_i=1}^{\mathfrak{N}_{i,k}}\frac{\left(lnt_{{\rm r},n_i}-\xi_{i_{\rm opt}}\right)^2}{\tau_{i_{\rm opt}}^3} - \frac{1}{\tau_{i_{\rm opt}}} + I_{{\rm F},i}\left(T_{\Gamma_k}\right)\right. \\ \left.\times\left(\frac{\left(lnt_{{\rm r},\mathfrak{N}_{i,k}+1}-\xi_{i_{\rm opt}}\right)^2}{\tau_{i_{\rm opt}}^3} - \frac{1}{\tau_{i_{\rm opt}}} + \frac{\sum_{j=1}^{5}\dfrac{jb_jr\left(lnt_{{\rm r},\mathfrak{N}_{i,k}+1}-\xi_{i_{\rm opt}}\right)/\tau_{i_{\rm opt}}^2}{\left[1+r\left(lnt_{{\rm r},\mathfrak{N}_{i,k}+1}-\xi_{i_{\rm opt}}\right)/\tau_{i_{\rm opt}}\right]^{j+1}}}{\sum_{j=1}^{5}\dfrac{b_j}{\left[1+r\left(lnt_{{\rm r},\mathfrak{N}_{i,k}+1}-\xi_{i_{\rm opt}}\right)/\tau_{i_{\rm opt}}\right]^{j}}}\right)\right] = 0 \end{cases} \tag{16}$$

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simulation and sequential simulation methods applied for short-term reliability evaluations, in this subsection, we compare these three methods by discussing the unavailability under different lead timescales of a single component which operates with binary Markovian states, that is, W and F states. The relevant state transition rates are assumed to be constant for all these concerned lead timescales.

First, according to the Chapman–Kolmogorov equations, the closed-form expressions of instantaneous state probabilities of the component can be given as

$$\begin{bmatrix} P_{\mathrm{W}}(T_{\mathrm{L}}) \\ P_{\mathrm{F}}(T_{\mathrm{L}}) \end{bmatrix} = \frac{1}{\lambda+\mu} \times \begin{bmatrix} \mu+\lambda \mathrm{e}^{-(\lambda+\mu)T_{\mathrm{L}}} & \mu-\mu \mathrm{e}^{-(\lambda+\mu)T_{\mathrm{L}}} \\ \lambda-\lambda \mathrm{e}^{-(\lambda+\mu)T_{\mathrm{L}}} & \lambda+\mu \mathrm{e}^{-(\lambda+\mu)T_{\mathrm{L}}} \end{bmatrix} \times \begin{bmatrix} P_{\mathrm{W}}(0) \\ P_{\mathrm{F}}(0) \end{bmatrix} \tag{18}$$

where $P_{\mathrm{W}}(T_{\mathrm{L}})$ and $P_{\mathrm{F}}(T_{\mathrm{L}})$ denote the component instantaneous W and F state probability at T_{L}, respectively, whereas λ and μ denote the component failure rate and repair rate, respectively.

Next, in conventional short-term reliability evaluations of the power system, since a concerned T_{L} is normally small (typical values as 10 min–10 h), it is accepted that no repair is conducted during such a short interval, thus, the unavailability is generally replaced with the outage replacement rate [22] which is numerically equal to $P_{\mathrm{F}}(T_{\mathrm{L}})$ by setting $\mu = 0$ in (18). Specifically, given the component operating in the W state at $t = 0$, the outage replacement rate at T_{L} is given by

$$\mathrm{ORR}(T_{\mathrm{L}}) = 1 - \mathrm{e}^{-\lambda T_{\mathrm{L}}} \tag{19}$$

Finally, we consider the average unavailability – another useful statistic value affecting the component reliability [36] – which is denoted by $U_{\mathrm{avg}}(T_{\mathrm{L}})$ in this paper and can be obtained from (18) as

$$U_{\mathrm{avg}}(T_{\mathrm{L}}) = \frac{1}{T_{\mathrm{L}}(\lambda+\mu)} \int_0^{T_{\mathrm{L}}} \begin{bmatrix} P_{\mathrm{W}}(t) & P_{\mathrm{F}}(t) \end{bmatrix} \times \begin{pmatrix} \lambda[1-\mathrm{e}^{-(\lambda+\mu)t}] \\ \lambda+\mu \mathrm{e}^{-(\lambda+\mu)t} \end{pmatrix} \mathrm{d}t \tag{20}$$

Now, it is worth underlining that a snapshot of the component reliability at T_{L} can be described by (18) or (19) which can both be obtained either by their individual analytical expressions (seems only feasible for simple systems) or by non-sequential simulation, whereas $U_{\mathrm{avg}}(T_{\mathrm{L}})$ can be considered as an indicator accounting for a sort of average reliability within lead time T_{L}. $U_{\mathrm{avg}}(T_{\mathrm{L}})$ can be obtained either by its analytical expression (also seems only possible for simple systems) or by simulation only in sequential manner in principle. The relationship between $U_{\mathrm{avg}}(T_{\mathrm{L}})$ and $P_{\mathrm{F}}(T_{\mathrm{L}})$ has been studied in many literatures, such as [4, 36].

Now, we compare the sequential and analytical methods by calculating $U_{\mathrm{avg}}(T_{\mathrm{L}})$ with different magnitudes of μ while $T_{\mathrm{L}} = 1$ h and $\lambda = 1$ (1/h). The simulated magnitudes of

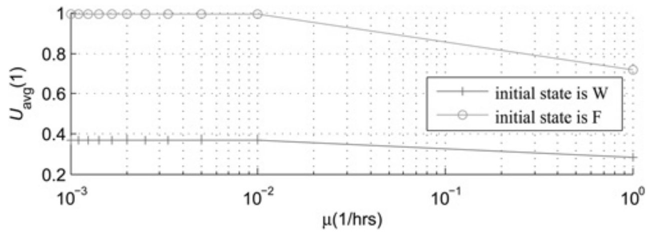

Fig. 3 *Component $U_{avg}(1)$ simulated with the proposed method against μ under logarithmic abscissa axis*

Table 1 Component U_{avg}s obtained with different methods under different magnitudes of μ

	1	0.1	0.01	0.001
ANA1, %	0.2838	0.3577	0.3668	0.3678
ANA2, %	29.61	2.83	0.28	0.028
2*CSDS/	0.25%	0.38%	0.67%	3.07%
SSTS	(3.4×10^{-3})	(1.2×10^{-2})	(3.9×10^{-2})	(0.12)
2*Adaptive	0.17%	0.21%	0.23%	0.24%
IS	(1.5×10^{-3})	(2.2×10^{-3})	(2.3×10^{-3})	(2.8×10^{-3})

The component is assumed to initially operate in W state.

$U_{\mathrm{avg}}(1)$ against different magnitudes of μ are plotted in Fig. 3. It is worth underlining that each simulated $U_{\mathrm{avg}}(1)$ is almost equal to the corresponding analytical solution obtained from (20), thus the analytical solution for each experiment point (U_{avg}, μ) will not be shown for conciseness. However, we select four points from Fig. 3 and list the corresponding numerical results in Table 1 for comparison. $U_{\mathrm{avg}}(1)$ computed by (20) is designated as analytical method (ANA)1, which is taken as the benchmark for comparison. As the repair rate is generally neglected in short-term reliability evaluation, we also compute $U_{\mathrm{avg}}(1)$ with $\mu = 0$ and list the percentage error in the second row, designated as ANA2, with respect to the result of ANA1. It can be noted that the error of $U_{\mathrm{avg}}(1)$ is 29.61% when $\mu = 1$ is neglected, and it decreases with μ. The results obtained with different simulation methods are also listed in the form of percentage error for comparison. Each number in parenthesis is the corresponding co under the simulation runs of 10^5. It can be concluded that the sequential simulation methods can yield $U_{\mathrm{avg}}(t)$ without bias, which suggests that the sequential simulation method is useful to give average reliability information within a short lead time for short-term reliability study, especially for complicated systems.

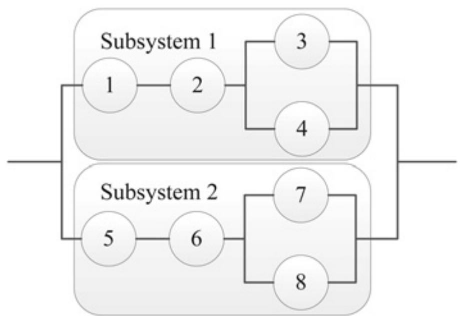

Fig. 4 *Configuration of a hypothetical small series–parallel system*

Table 2 Failure rates and parameters of the lognormal distributions counting for repair time of the components in the hypothetical series–parallel system

	λ (1/h)	ξ	τ
components 1 and 5	0.001	−0.0549	1.0510
components 2 and 6	0.002	−0.0607	1.1834
components 3 and 7	0.003	−0.5141	1.1516
components 4 and 8	0.004	0.7305	1.1062

Table 3 Simulated $U_{avg}$of the series–parallel system with assumption of lognormal distribution for repair time of components

	CSMC	Adaptive IS
U_{avg} (co = 0.05)	1.951×10^{-4}	2.025×10^{-4}
CPU time, s	64014	93.75
U_{avg} (co = 0.04)	2.135×10^{-4}	1.917×10^{-4}
CPU time, s	1.45×10^{5}	98.8

5.5.2 Glance at efficiency: The last subsection explains the virtues of sequential simulation methods by applying them to a simple Markov process. This subsection further explains the proposed algorithm by applying it to a hypothetical series–parallel system, as shown in Fig. 4, which operates in semi-Markov process. The studied system consists of two identical subsystems working in parallel each composed of four components. Subsystem 1 for example functions if and only if both components 1 and 2 in series function and at least one of components 3 and 4 in parallel functions. Each component identically operates with binary Markovian states. Table 2 lists the component original failure rates and parameters of the lognormal distribution used to describe repair time which are extracted from [27].

System average unavailability within a lead time of one hour is simulated for illustration. It is obvious that components 1, 2, 5 and 6 are more important to ensure the operational reliability of the system. To decrease the number of parameters to be solved, we fix τ of the repair time distribution for each component, and as a result (16) can also be simplified to be solved. As expected, λ_1, λ_2, λ_5 and λ_6 are reasonably increased after Step iv of the proposed method, and the final distorted solutions are $\lambda_{1_{opt}} = 0.1881$, $\lambda_{2_{opt}} = 0.5842$, $\lambda_{5_{opt}} = 0.2161$ and $\lambda_{6_{opt}} = 0.505$, respectively. Simulated results of U_{avg} combined with CPU times are listed in Table 3. In addition, the coefficient of variance, denoted by co, are also shown in parenthesis. It can be noted from the results that the proposed method can improve simulation efficiency considerably. The CPU time by the proposed method is shortened to 1/682 and 1/1467 of that by the CSMC method under the conditions of co = 0.05 and co = 0.04, respectively.

6 Application results

To evaluate the accuracy and efficiency of the proposed method, tests are conducted on R-RBTS [37] which consists of 10 units totalising 240 MW of installed capacity with a peak load of 185 MW. Three indices, viz. EENS, LOLE and LOLF, are calculated for illustrations. All the following calculations are coded in Matlab and conducted on the platform of AMD Athlon II X4 640 3.00 Hz. AC power flow and AC optimal power flow based on Matpower are called for to analyse the sampled system states. Ratio of figures of merit [38], denoted by Ef, is introduced as (21) to measure efficiency gain of the proposed method with respect to the CSMC method

$$\mathrm{Ef} = \frac{t_c\sigma_c^2}{t_{CE}\sigma_{CE}^2} \tag{21}$$

where t_c and t_{CE} are the CPU times and σ_c and σ_{CE} are the variances of reliability indices under the CSMC method and the proposed method, respectively.

All the components are assumed to initially work in normal state. The efficiency gain of the proposed method is firstly investigated under different load levels in the two scenarios of repair time distributions. Then, the system reliability within different lead timescales and different load levels are studied.

6.1 Efficient gain studies

The efficiency gain studies are conducted under four different cases where the lead time is fixed to one hour:

(1) Peak load level with lognormal distribution describing component repair time.
(2) Ninety percent of peak load level with lognormal distribution describing component repair time.
(3) Peak load level with Weibull distribution describing component repair time.
(4) Ninety percent of peak load level with Weibull distribution describing component repair time.

For the sake of impartial comparison, each of the following results is obtained by averaging 20 replicas under exactly the same setting of condition as follows: $|\mathbb{S}| = |\mathbb{S}'| = 600$, $|\mathbb{S}_{IS}| = 100\,000$. As the simulation with the CSMC method to obtain small co is very time-consuming, we limit the total number of simulation runs designated as N with the CSMC method to 500 000 for all the cases.

The original parameters of the lognormal distribution used in this paper can be derived as (22) through the method of moments in terms of the exponential distribution assumption for each component repair time in the original dataset

$$\begin{cases} \xi = -\ln\ \mu - 0.5\ \ln 2 \\ \tau = \sqrt{\ln\ 2} \end{cases} \tag{22}$$

where μ is the repair rate.

The simulation results for Case 1 are listed in Table 4 from which it can be noted that the co (in brackets) reduces to 0.085 under the CSMC method, whereas in comparison, that under the proposed method reduces to 0.01. The corresponding

Table 4 Simulated indices under the condition of lognormal distribution describing component repair time and peak load level

	EENS, MWh	LOLE, h	LOLF, occ./h
CSMC (0.085)	2.54×10^{-3}	2.23×10^{-4}	4.98×10^{-4}
adaptive IS (0.01)	2.73×10^{-3}	2.31×10^{-4}	4.66×10^{-4}

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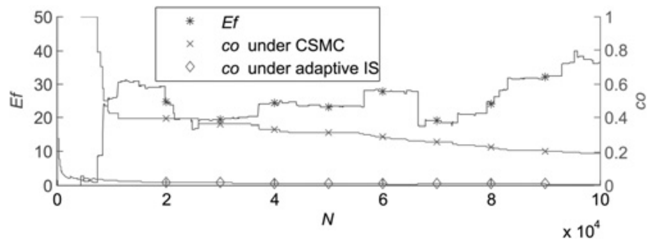

Fig. 5 *co and Ef against the number of simulation runs for R-RBTS under the conditions of lognormal distribution describing component repair time and peak load level*

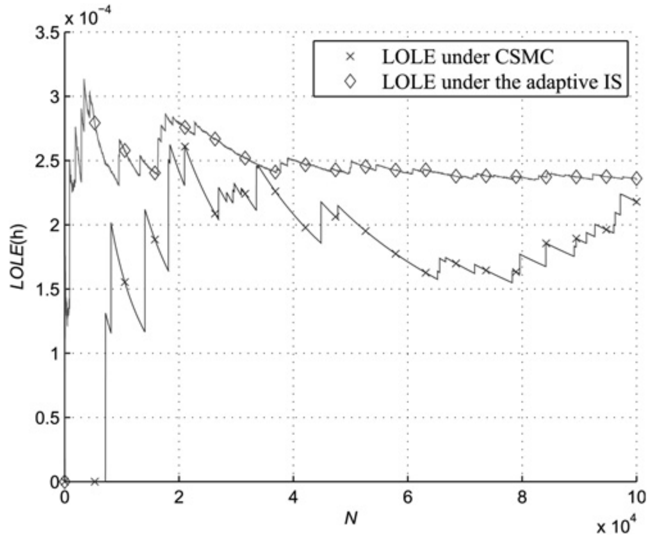

Fig. 6 *LOLE against the number of simulation runs obtained with the CSMC and the adaptive IS methods tested on R-RBTS, under the conditions of lognormal distribution describing component repair time and peak load level*

relationship between Ef and the number of simulation runs as well as that between co and the number of simulation runs are plotted together in Fig. 5, which shows that the proposed method is more efficient than the CSMC method. Also, there is a plot hereafter showing the estimate of LOLE with respect to the number of simulation runs, which clearly illustrates the increased convergence speed of the proposed method (see Figs 6, 8, 10 and 12). It is worth mentioning that as the system failure event is rare in such a short T_L, reliability metrics as LOLE obtained by the CSMC method need a huge number of simulation runs, for example, 100 000 in our numerical studies, to converge (Fig. 6).

The simulation results for Case 2 are listed in Table 5. It can be noted that the coefficient of variances under both methods are larger than that in the case of the peak load level; however, the results obtained with the CSMC method are much less credible because of its high co. By observing

Table 5 Simulated indices under the condition of lognormal distribution describing component repair time and 90% of peak load level

	EENS, MWh	LOLE, h	LOLF, occ./h
CSMC (0.2781)	6.72×10^{-4}	1.61×10^{-4}	3.64×10^{-4}
adaptive IS (0.0124)	6.80×10^{-4}	1.61×10^{-4}	3.46×10^{-4}

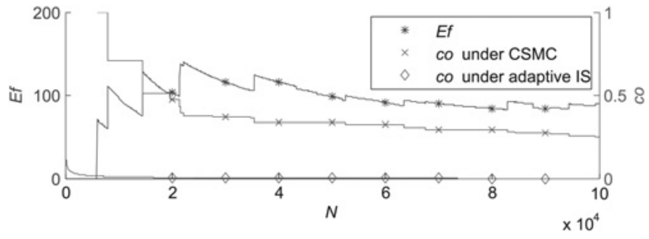

Fig. 7 *co and Ef against the number of simulation runs for R-RBTS under the conditions of lognormal distribution describing component repair time and 90% of peak load level*

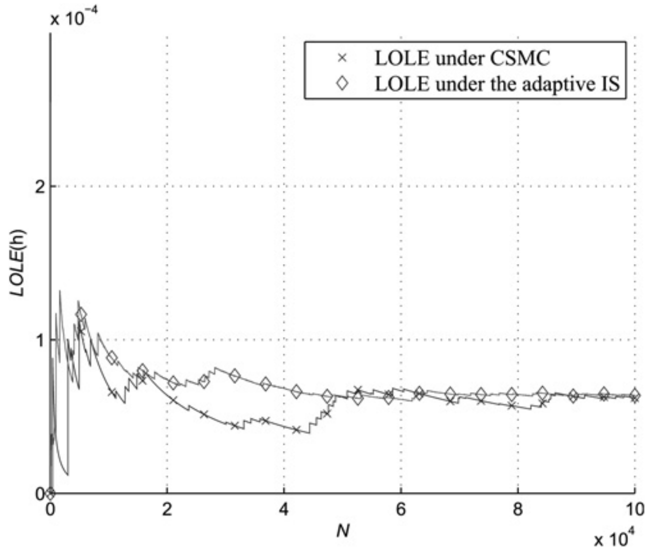

Fig. 8 *LOLE against the number of simulation runs obtained with the CSMC and the adaptive IS methods tested on R-RBTS, under the conditions of lognormal distribution describing component repair time and 90% of peak load level.*

Table 6 Simulated indices under the condition of Weibull distribution describing component repair time and peak load level

	EENS, MWh	LOLE, h	LOLF, occ./h
CSMC (0.088)	2.313×10^{-3}	2.029×10^{-4}	4.54×10^{-4}
adaptive IS (0.008)	2.563×10^{-3}	2.097×10^{-4}	4.439×10^{-4}

the Ef in Figs. 5 and 7, it can be noted that the Ef increases by approximately 1.5 times from 40 in the scenario of peak load level to 100 in the scenario of 90% of peak load level at the 100 000th simulation. It is validated that the proposed method becomes even more efficient than the CSMC method as the system reliability increases.

For Case 3 and Case 4, as exponential distribution is a special case of Weibull distribution, the parameters of Weibull distribution used for simulation can be readily found to be $\tau = 1$ and $\xi = 1/\mu$. The simulation results in the two situations are listed in Tables 6 and 7, respectively, and the corresponding Ef and co against the number of simulation runs are potted in Figs. 9 and 11, respectively. It can be noted that similar conclusions as those in Case 1 and Case 2 can be drawn, except that Ef under the Weibull

Table 7 Simulated indices under the condition of Weibull distribution describing component repair time and 90% of peak load level

	EENS, MWh	LOLE, h	LOLF, occ./h
CSMC (0.091)	6.29×10^{-4}	1.49×10^{-4}	3.48×10^{-4}
adaptive IS (0.01)	7.02×10^{-4}	1.67×10^{-4}	3.47×10^{-4}

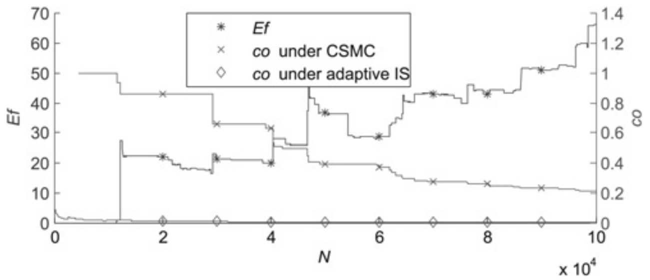

Fig. 9 *co and Ef against the number of simulation runs for R-RBTS under the conditions of Weibull distribution describing component repair time and peak load level*

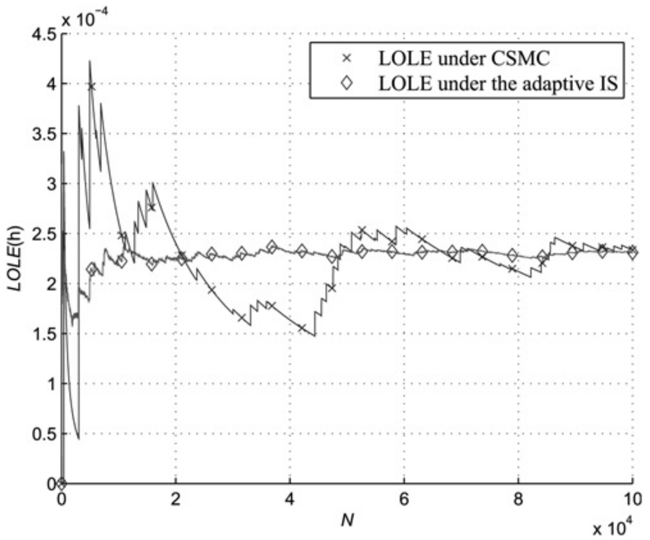

Fig. 10 *LOLE against the number of simulation runs obtained with the CSMC and the adaptive IS methods tested on R-RBTS, under the conditions of Weibull distribution describing component repair time and peak load level*

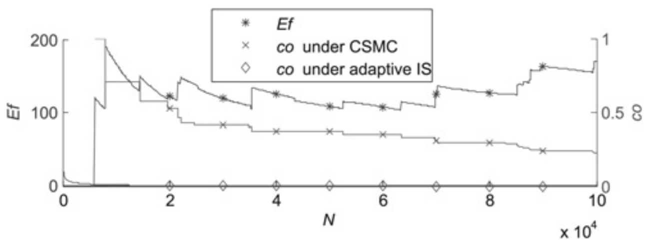

Fig. 11 *co and Ef against the number of simulation runs for R-RBTS under the conditions of Weibull distribution describing component repair time and 90% peak load level*

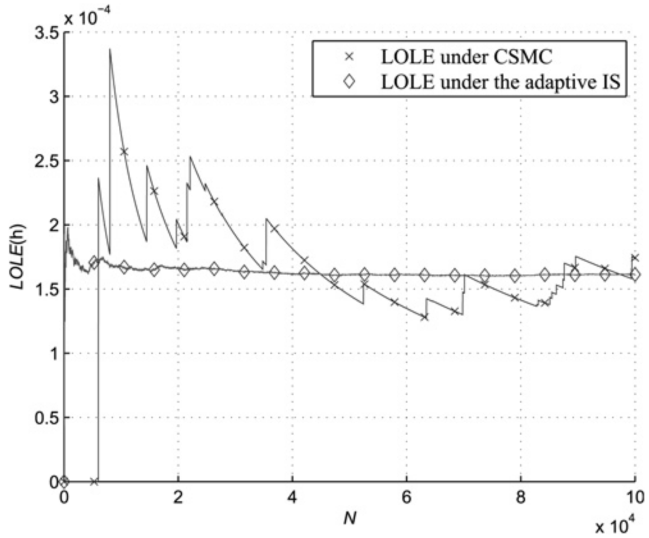

Fig. 12 *LOLE against the number of simulation runs obtained with the CSMC and the adaptive IS methods tested on R-RBTS, under the conditions of Weibull distribution describing component repair time and 90% peak load level.*

distribution scenario is greater than that under the lognormal distribution scenario at the same load level.

It can be concluded that the efficiency gain of the proposed method is robust to system load levels and distributions describing the component repair time. Also, it is interesting to find that the magnitudes of the simulated indices are sensitive to the system load level but seem irrelevant to the distributions used to describe component repair time, and the possible reason may be that the mean and standard deviation parameters of the used distributions are equal, respectively.

6.2 Effects of load level and lead timescale on system reliability

We investigate indices associated with different lead timescales under different load levels. Weibull distribution is assumed to describe component repair time. The results are plotted in Fig. 13. As for the case of the lognormal distribution for component repair time, according to the

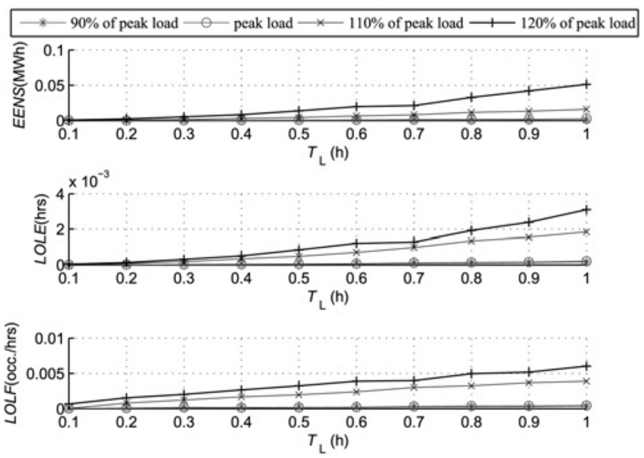

Fig. 13 *Relationships between indices and lead time scales for R-RBTS under different load levels*

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investigations conducted in the previous subsection, similar results are obtained and not reported for conciseness.

It can be noted that the indices monotonically increase with the lead timescale, and much more rapid increment is observed in the case of a higher load level. The proposed method is useful to monitor the system operating pressure under different requirements for the lead timescale.

7 Conclusion

The short-term reliability evaluation of non-homogeneous composite power systems is of practical significance for modern electric power utilities. The sequential Monte Carlo method can yield frequency and duration indices without bias in a straightforward fashion, however, the corresponding computational cost is prohibitively expensive. In this paper, an adaptive importance sampling is proposed to ameliorate this situation. Lognormal and Weibull distributions are used to describe component repair time. Through the case studies conducted on a reinforced Roy Billinton reliability test system, it suggests that the ratio of the figure of merit Ef with respect to the crude sequential Monte Carlo in the case of peak load is greater than 40 and 70 in the scenarios of the two component repair time distributions, respectively, and increases approximately by 1.5 times to 100 and 170 in the case of 90 percent of peak load, respectively. It validates that the proposed method improves simulation efficiency considerably and is robust to the system load levels and component repair time distributions, and more efficient in the case of Weibull distribution. Thus, the proposed method is useful to monitor the system operating pressure under different requirements for the lead timescale.

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